A PRIMER and WORKBOOK for UCI CASHFLOW ANALYSIS STUDENTS

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This workbook is in response to many requests, over many years, from many *Cashflow Analysis* financial planning students, for additional problems with which to hone their analytical and technical skills.

Unfortunately, many students are hampered by inadequate or very rusty math skills, particularly in some basic areas. Therefore the first few pages are devoted to getting started in using the HP–12c calculator and then to brushing up a few long-unused math skills.

Problems in the workbook generally follow the subjects in the text, chapter by chapter. But occasionally, a problem may revert to a prior subject for additional amplification and continued reinforcement.

Using This Workbook

There is a right way and a wrong way to use this workbook. If a student attempts a problem, fails to obtain the right answer and then simply moves on to the next exercise, most of the benefit of the workbook will be lost.

Getting the wrong answer is an opportunity to learn. It's important not only to discover how the correct answer was obtained, but also to compare this solution with the approach which delivered an incorrect answer.

The benefit to be realized from doing a large number of problems is that, over time, underlying principles are extracted and universal concepts begins to form. This is when a student says "Oh, now I see!"

When this wonderful learning moment occurs, the student takes permanent ownership of the concept, stripped of all its incidentals, and is able to apply it to any number of similar situations in the future.

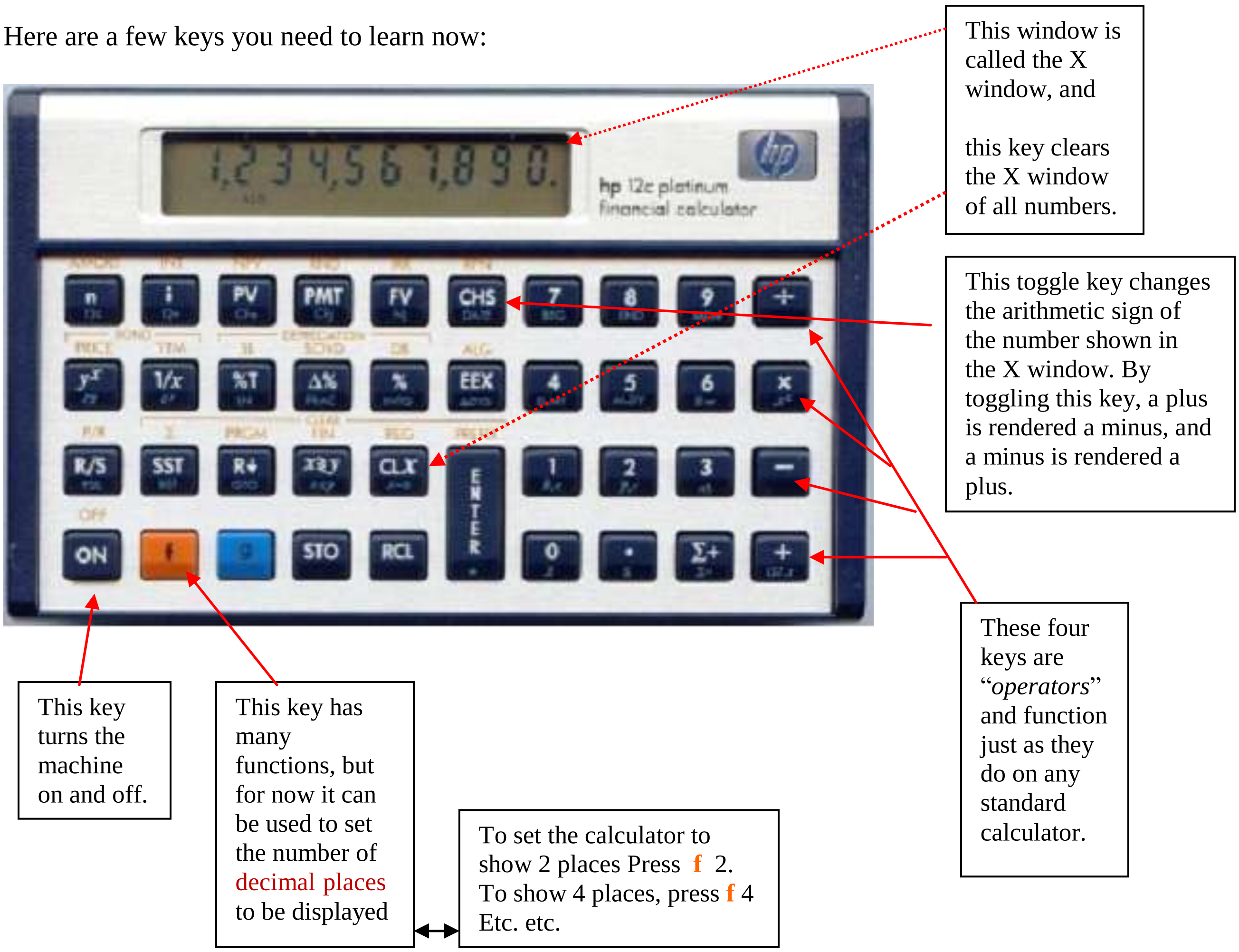
This is the moment when satisfaction and confidence build; it is the moment when the student and the concept become one.

Shaking Hands with the HP–12C

The *Cashflow Analysis* course and this workbook use Hewlett Packard's HP–12C calculator.

If you have just recently acquired your calculator, it may seem hopelessly complicated. But rather than attempt to learn all the keys and functions in one sitting, simply focus initially on learning a few fundamentals; all other functions will fall into place in due time, as the need arises.

Here are a few keys you need to learn now:



A shorthand used in the text for the location of keys is described in terms of (#row, #key). The key FV, for example, is located at (1, 5) … indicating the 1st row, 5th key from the left. The location (3, 5) locates the CLX key.

If you press f before you press another key, the function on the top of the key (in orange) is enabled. If you press g before you press another key, the function on the lower portion of the key (in blue) is enabled.

1. Reverse Polish Notation

This platinum edition of the HP-12C offers two operating systems: the first is the familiar algebraic system. In the algebraic system the logic follows the path x + y = z, or 2 + 3 = 5. By pressing **f** **ALG** the algebraic system is enabled. (Key **f** (2,6)

The second operating system is called Reverse Polish Notation (RPN). This is a very useful system and has many advantages over the algebraic system. This system is enable by pressing **f** **RPN .** (Key **f** (1,6)

Set your calculator to use RPN by pressing **f** and then key (1,6). You will see the letters RPN in the lower left corner of X window.

2. The Logic of RPN

Fortunately, you already know how to use RPN because you learned it in elementary school when your teacher asked you to go to the board and write the number 6.
Then she directed you to write the number 3 beneath the 6.

Your blackboard looked like: 6
<u>3</u>

Then your teacher told you what to do with these two numbers: divide, multiply, subtract or add.
She told you which *operators* to apply *after* (post) you had written the numbers.

<u>That's exactly how RPN works</u>: you first enter the values and then tell the calculator what to do with them by pressing an *operator* key. If you divided, you got 2 for an answer; if you multiplied, you got 18; if you subtracted you got 3, and if you added you got 9. Here's the same exercise on the HP–12C:

Keystrokes	Display	Keystrokes	Display	Keystrokes	Display	Keystrokes	Display
f REG	0.00	f REG	0.00	f REG	0.00	f REG	0.00
6 Enter	6.00	6 Enter	6.00	6 Enter	6.00	6 Enter	6.00
3	3	3	3	3	3	3	3
÷	2.00	X	18.00	–	3.00	+	9.00

RPN is that simple!

3. Chaining on the HP-12C

One of the great conveniences of the RPN system is the ability to ***chain*** calculations. Buy "chain" we mean to progress from the result of one calculation to the next without having to enter the results of the previous calculation.. Here's a simple example:

Total the following numbers: 10, 15, -9, 45, and 3.
Then multiply the result by 4.
Then divide the result by 1.5.

REG is located above the <u>CLX</u> key at location (3,5).
f REG clears all registers in the calculator (except programming registers).
It is important to clear all registers before you begin a new calculation because the HP 12c will retain values in its registers ***even after you turn it off.***

f REG	0.00
10 Enter	10.00
15 +	25.00
9 –	16.00
45 +	61.00
3 +	64.00
4 X	256.00
1.5 ÷	170.67

Notice that the *operator* (+) comes *after* (post) the entry of the values. This explains the "reverse" part of "Reverse Polish Notation."

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4. Working with Exponents and Roots

Read the bottom of page 1-7 and the top of page 1-8 in the text. Then return here.

When we use an *exponent*, we multiply a number by itself for as many times as is indicated by the exponent.

For example:

What is the value of 10^5 ?

This small number is the *exponent*. It can be an integer, a decimal or a fraction.

We could also ask: "What is the value of 10 raised to the 5^{th} *power*?"
The exponent in this case is <u>5</u>, indicating that we need to multiply 10 by itself <u>5</u> times;

<u>1</u> <u>2</u> <u>3</u> <u>4</u> <u>5</u>
10 x 10 x 10 x 10 x 10 = 100,000.00

A simpler way is to use the calculator's exponent key $\boxed{y^x}$ ((2,1):

f REG	0.00
10 Enter	10.00
5 $\boxed{y^x}$	100,000.00

You probably already know that any number raised to the zero (0) power is *always* equal to 1.

e.g. $\frac{10{,}000}{x^{0}} = \frac{10{,}000}{1} = 10{,}000$

5. Determining the Root of a Number

Determining the root of a number is the opposite of raising a number to a power.

For example, we could ask: "What is the 5^{th} ***root*** of 100,000?"
Or we could ask "What number multiplied by itself *5 times* will yield 100,000?"

In order to find the ***root*** of a number we will create an exponent in the form of a fraction; the numerator will be 1, and the denominator will be the root we are seeking. For example,

$100{,}000^{\frac{1}{5}}$

It's easy to see that $\frac{1}{5}$ = 0.2. So the exponent may also be written as a decimal.

The fifth root of 100,000 is written as $100{,}000^{0.2}$ or $\sqrt[5]{100{,}000}$

This symbol $\sqrt{}$ is called a 'radical.'

f REG	0.00
100,000 Enter	100,000.00
0.2 $\boxed{y^x}$	10.00

Therefore: $\sqrt[5]{100{,}000} = 10$
This shows that multiplying 10 by itself five times over will deliver 100,000.

If we were looking for the 6^{th} root, we would have entered:

f REG	0.00
100,000 Enter	100,000.00
6 1/x	0.17
y^x	6.81

Notice that we *chained* steps in this example in order to determine the value of 100,000 raised to the 1/6 power. The key 1/x is at location (2,2)

This is a good time to explain that the HP-12c always calculates the answer to an operation using 10 digits. Therefore the answer to **6** 1/x in the table above is not 0.17 as displayed, but 0.166666667. You have your calculator set to display only 2 places, therefore the X window shows only 0.17, but continues to use 0.166666667.

If you reset the calculator to show 9 decimal places (**f**, 9), and repeat the same operation, you will see the difference. The calculator uses all 10 digits in all its computations r*egardless of how many decimal places it is set to show..*

For this reason, it is always more accurate to *transfer* values from one register to another rather than reading (lifting) the value and using the keypad to re-enter the shown value into the destination register. In solving problems, determine the correct answer as accurately as possible – only then round up the answer, if appropriate.

5. Observing *Precedence*

What is the value of 9 + 7 x 8 ?
Two results are possible:

9 + (7 x 8) = 65, or
(9 + 7) x 8 = 128.

The correct answer depends upon which operation(s) you perform first.
The math community has established the order, or *precedence*, in which operations of more complex formulas should be carried out. The order is easy to remember:

Please **E**xcuse **M**y **D**ear **A**unt **S**ally

This mnemonic tells you to perform the operations in the following order:

1	Parentheses
2	Exponents
3	Multiply
4	Divide
5	Add
6	Subtract

Suppose you had the following calculation:

$$\mathbf{X} = 7*[\frac{6.0}{1.5}]^{\mathbf{2}} + 5 - 2$$

In following the rules of *precedence* you would first perform all the functions within the parentheses, then all functions attached to the parentheses, then the remaining operations in the order proscribed: **PEMDAS**:

	f REG	0.00
Parentheses	**6Enter**	6.00
	1.5 ÷	4.00
Exponents	**2** y^x	16.00
Multiply	**7 x**	112.00
Add	**5 +**	117.00
Subtract	**2 –**	115.00

This operation is within the parenthesis

In a word, you always start with the innermost part of the calculation and then work to the outside.

6. Working with Fractions and Mixed Numbers

Many students who return to school after many years have forgotten some of the rules concerning the handling of fractions.

Rule 1: In order to add or subtract fractions, their denominators { $\frac{\text{Numerator}}{\textbf{Denominator}}$ } must be the same. If the denominators are the same, add (or subtract) the numerators but retain the denominator.

For example: $\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$ $\quad$ $\frac{6}{7} - \frac{2}{7} = \frac{4}{7}$

In these examples, we simply added the numerators and retained the same denominator.

Rule 2: When the denominators are not the same, you must find a *common denominator*. A common denominator means a denominator into which all the different denominators can be divided evenly. To convert a fraction into an equal fraction which uses the common denominator, first divide the denominator into the chosen common denominator: then multiply the result times the numerator of the same fraction.

For example: $\frac{1}{7} + \frac{2}{3} = \frac{3}{21} + \frac{14}{21} = \frac{17}{21}$ The number 21 is the chosen common denominator.

You can always find the a common denominator by multiplying the denominators of each fraction together, although a smaller number can sometimes serve as a common denominator.

For example: $\frac{1}{4}+\frac{3}{8}=\ \frac{2}{8}+\frac{3}{8}=\frac{5}{8}$ **OR** $\frac{1}{4}+\frac{3}{8}=\ \frac{8}{32}+\frac{12}{32}=\ \frac{20}{32}=\frac{5}{8}$

<u>Rule 3</u>: To multiply fractions, multiply all the numerators *and* then all the denominators:

$$\frac{2}{3}\text{x}\frac{3}{4}x\frac{7}{8}=\frac{42}{96}\ .$$

This fraction can be ***reduced*** to a simpler fraction without changing its value by dividing the numerator and denominator by the same number, In this case, <u>6</u>.

Hence, $\frac{42}{96}=\frac{7}{16}$.

It can't be reduced further because there is no larger divisor which will divide into 7 and 16 evenly.

<u>Rule 4</u>: When a fraction is divided by an integer, simply multiply the denominator of the fraction by the integer.

E.g. divide $\frac{\frac{3}{4}}{3}$ $\quad \frac{3}{4*3}=\frac{3}{12}=\frac{1}{4}$

<u>Rule 5</u>: To divide a fraction by another (second) fraction, **invert** the second fraction and multiply the numerators of each fraction and the denominators of each fraction.

E.g.: Divide $\frac{3}{4}$ by $\frac{5}{8}$ $\quad \frac{3}{4}x\frac{8}{5}\ =\ \frac{24}{20}=\frac{6}{5}=1.2$

Mixed numbers consist of a whole number (an integer) and a fraction, such as $4\frac{2}{3}$.

To convert a mixed number to an ("improper") fraction multiply the denominator of the fraction by the whole number and add the numerator of the fraction. Retain the denominator.

For example:

$$4\frac{2}{3}=\frac{12+2}{3}=\frac{14}{3}$$

This is an *improper* fraction because its numerator is larger than its denominator.

7. Reciprocals

The *reciprocal* of a number is equal to 1 divided by the same number. Therefore the reciprocal of $2 = \frac{1}{2}$. The reciprocal of $\frac{1}{2} = \frac{1}{\frac{1}{2}} = 1 * \frac{2}{1} = 2$.

Remember that to divide by a fraction, you invert the fraction and multiply.

To find the *reciprocal* of a fraction, turn it upside down.

The reciprocal of $\frac{4}{5} = \frac{5}{4} = 1.25$

The key on the HP 12-C located at (2,1) converts any number or fraction to its reciprocal.

8. The Time Value of Money

When we say that "money has a time value," we mean that the value of money changes depending upon the time to receive or expend it.

The basic formula to express the time value of money is:

$$PV = \frac{FV}{(1+i)^n}$$

PV stands for present value, sometimes called *present worth*.

FV stands for *Future Value*.

The symbol i can represent an interest rate or a discount rate. *Interest rates* move the present value of money forward in time to create a future value. The same symbol can also represent a *discount rate* which moves a Future Value backward in time to express a present value.

The exponent n stands for the number of time periods between the PV and the FV.

When there are no periodic payments involved in a cashflow problem, or when the cashflow payments are *exactly* the same, the registers (1,1) to (1,5) can be used.

n	i	PV	PMT	FV

9. Some Basic Examples

> You have accepted a promissory note as part of a consultation fee. The note promises to pay you $3,500 in 3 years. What is its value to you today?

Ans. Let's assume that if you had the cash in hand today, the next best investment you could find (of similar risk) would yield 8% per year. Therefore 8% is your *opportunity cost* of funds, and we will use this rate to determine the present value of the future receipt of $3,500:

$$PV = \frac{FV}{(1+i)^n} \qquad PV = \frac{\$3{,}500}{(1+.08)^3} = \frac{\$3{,}500}{1.26...} = \$2{,}778.41.$$

The three dots after the 6 is called an *ellipsis* and indicates (in math) there are digits which follow but are not shown. To show the number used by the calculator, press f Prefix. Prefix is on top the ENTER button.

We can also use the calculator's "horizontal" registers to solve this problem:

n	i	PV	PMT	FV
3	8	?	-	3.500
		-2,778.41		

Interest rates and discount rates are ***always*** entered into the **i** register as percentages, never as decimals. [1]

If the time to receipt of the $3,500 were pushed farther into the future, say 5 years instead of 3, the PV of the future cashflow would be considerably less:

$$PV = \frac{\$3{,}500}{(1+.08)^5} = \frac{\$3{,}500}{1.47...} = \$2{,}382.04$$

Determining the PV of a FV is called *discounting*. In this case, the PV of $3,500 to be received 5 years in the future has a Present Value of $2,382.04 *when discounted* at 8% per year.

Therefore, you can make mental note of the fact that as the time to receipt of a cashflow increases, the Present Value decreases. This is true of the discount rate (i) as well: as the discount rate *increases* the Present Value *decreases*.

In other words, the Present Value varies inversely with the time to receipt (n) and also inversely with the discount rate (i).

10. Carrying a Present Value Forward in Time

It is often required to determine the Future Value of a sum which is invested today. In this case the symbol i stands for an *interest rate*.

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[1] This is not the case when using Excel functions. All rates must be entered as decimals.

$$FV = PV*(1+i)^n$$

Let's apply this simple formula to a real-life problem:

You have the sum of $2,778.41 available to you today. You can invest this sum @ 8.0% annual interest for 3 years. What will be the (future) value at the end of the 3rd year?

$$FV = PV*(1+i)^n$$

$$FV = 2,778.41 * (1+.08)^3$$

The asterisk here indicates a multiplication. But it is customarily omitted, and may be hereafter.

Following the rules of *precedence*, first determine the value of $(1+.08)^{3.}$

The first step is do everything inside the parentheses: 1+.08 = 1.08
Then, raise the result, 1.08, to the 3rd *power*.
After that, multiply the result by the value 2,778.41.
The result will be the Future Value.
Here are the calculator steps:

f REG	0.00
1 Enter	1.00
.08 +	1.08
3 y^x	1.26
2778.41 x	3,500.00

Clears the calculator

The calculator makes this an easy task:

n	i	PV	PMT	FV
3	8	–2,778.41	-	?
				3.500

See Page 1-9 in the text for an explanation of why $2,778.41 is entered as a negative number.

11. Nominal vs. Effective Interest Rates

Interest rates can be expressed in a number of ways. The most common way is to express a rate as an annual *nominal rate*. ('nominal' means in name only.).
Effective rates are created as the result of *compounding*.

For example, if you opened a bank account with a deposit of $1.00 which pays 4.0% per annum but does not compound the rate at all, the balance (FV) of you account at the end of one year would be:

$$FV = PV^*(1+i)^1$$
$$FV = \$1.00 * (1+0.04) = \$1.04$$

But the bank may compound this 4.0% rate quarterly. In that case, the balance of the account in one year would be:

$$FV = PV^*(1+\frac{.04}{4})^4$$
$$FV = \$1.00 * 1.0406 = \$1.0406$$

If the bank compounded its rate monthly, the result would be:

$$FV = PV^*(1+\frac{.04}{12})^{12}$$
$$FV = \$1.00 * 1.04074 = \$1.04074$$

If the bank compounded its rate daily (365 days), the result would be:

$$FV = PV^*(1+\frac{.04}{365})365$$
$$FV = \$1.00 * 1.04081 = \$1.04081$$

> After subtracting the initial $1.00, we can see that the interest per dollar increases as the number of compounding periods increases. The results are the effective rates:
>
> 0.0406 = 4.06%
> 0.04074 = 4.074%
> 0.04081 = 4.081%

Therefore we can say that the *effective* rate is the rate created by *compounding*.

To Be Continued